```
B2 2 3 0 GFET
CL 2 0 6F
.MODEL GFET GASFET(VTO = -2.5, BETA = 65U, VBI = .5,
ALPHA = 1.5, TAU = 10PS)
.MODEL GF1 GASFET(VTO = -2.5, BETA = 32.5U, VBI = .5,
ALPHA = 1.5)
.TRAN .0125NS INS
.PRINT TRAN V(3) V(2)
.PLOT TRAN V(3) V(2) (-1 4)
.END
```

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Letters

Comment on "Microwave Diffraction Tomography for Biomedical Applications"

THEODORE C. GUO, SENIOR MEMBER, IEEE, WENDY W. GUO, SENIOR MEMBER, IEEE, AND LAWRENCE E. LARSEN, SENIOR MEMBER, IEEE

In the above paper [1], and a subsequent publication [2], Bolomey *et al.* suggested a very useful technique of biological tomography which may provide a fast image reconstruction at low cost. In a more recent work [3], Bolomey also presented some interesting results of a reconstructed image of a pony kidney. The virtue of the suggested technique lies in the rapidity of data collection by modulated scattering. Here we would like to point out that more theoretical study is still needed on the interpretation of the reconstructed image. The algorithm of the technique is based on the identity

$$\tilde{E}(\alpha,\beta;z)=\tilde{E}(\alpha,\beta;z_0)\,e^{i(z-z_0)\sqrt{k_m^2-\alpha^2-\beta^2}} \tag{1}$$

where $\tilde{E}(\alpha,\beta;z)$ is the Fourier transformation (FT) of the electric field in the xy-plane and k_m is the wavenumber in the medium surrounding the target. Thus, from measurements of $\vec{E}$ in a plane at $z=z_0$, one may predict the field in another parallel plane by simply making an FT followed by an inverse FT. However, it must be pointed out that the above identity is valid only when both planes are outside the target (the pony kidney in this case) and on the same side of the target.

To illustrate this point, let us consider a scalar field of a point source located at $\vec{x}=0$. The field is given by $A(\vec{x})=\exp(ik_m r)/r$. A measurement of the field in the plane $z=z_0$ ($z_0>0$) will yield

$$A(x,y,z_0)=\frac{\exp\left(ik_m\sqrt{x^2+y^2+z_0^2}\right)}{\sqrt{x^2+y^2+z_0^2}}$$

of which the FT is

$$\tilde{A}(\alpha,\beta;z_0)=i\frac{\exp\left(i|z_0|\sqrt{k_m^2-\alpha^2-\beta^2}\right)}{\sqrt{k_m^2-\alpha^2-\beta^2}}$$

where, for $\alpha^2+\beta^2>k_m^2$, the square root is taken to have a positive imaginary part so that $\tilde{A}$ decays exponentially for large $\alpha^2+\beta^2$. Applying (1) and taking the inverse FT, one gets

$$A_R(x,y,z)=\frac{i}{2\pi}\int_{-\infty}^{\infty}d\alpha\int_{-\infty}^{\infty}d\beta \cdot e^{i(\alpha x+\beta y)}\frac{\exp\left[i\sqrt{k_m^2-\alpha^2-\beta^2}\,(|z_0|+z-z_0)\right]}{\sqrt{k_m^2-\alpha^2-\beta^2}} \tag{2}$$

where the subscript R denotes that it is a reconstructed field. Clearly, (2) gives the correct field as long as z and z_0 are both positive. But, if $z<0$ while $z_0>0$, the integral in (2) diverges because of the exponential factor. Of course, one may always truncate the integrand at $\alpha^2+\beta^2=k_m^2$, as it is always done in practical computation. Then the integration gives approximately $A_R(x,y,z)=-\exp(-ik_m r)/r$. Thus, the field reconstruction using (1) produces an outgoing wave in one side of the source and an incoming wave in the opposite side of the source, whereas the actual field is an outgoing wave in both sides of the source. For a continuous distribution of sources, $J(\vec{x})$, in a region V, the actual

Manuscript received July 6, 1983.

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0018-9480/84/0400-0473$01.00 ©1984 IEEE

field is

$$A = \iiint_V J(\vec{x}')\frac{\exp(ik_m|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}d\vec{x}'.$$

From measurements in an xy-plane, say, to the right of the region V, the reconstructed field will provide the valid field at every point to the right of V. But the reconstructed field in an xy-plane that passes through V is given by

$$A_R(\vec{x}) = \iiint_{V(z'<z)} J(\vec{x}')\frac{\exp(ik_m|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}d\vec{x}' + \iiint_{V(z'>z)} J(x')\frac{\exp(-ik_m|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}d\vec{x}'$$

which is quite different from the actual field $A(\vec{x})$.

The purpose of reconstructing the field is to construct the source distribution, which in turn provides desired information regarding the electromagnetic image of the scattering object. Even though the reconstructed field inside the scattering object is quite different from the actual field, the image obtained from the proposed technique must still represent some kind of dielectric characteristics of the scattering object. However, it is not clear exactly what characteristics it represents, and more theoretical study is needed in this regard.

Finally, we remark that active research on microwave biological imagery using a water-immersed microwave array [4] is also being performed at Walter Reed Army Institute of Research and the Johns Hopkins University Applied Physics Laboratory. Several approaches for obtaining the image from a set of limited measurements are being considered [5], [6]. Here, one of these approaches is briefly described. First, it is shown that the scattered field is equal to one that is produced by an effective charge-current distribution $(\rho_{\text{eff}}, \vec{J}_{\text{eff}})$, with

$$\rho_{\text{eff}} = -\nabla\cdot\left(\frac{\chi-\chi_m}{\chi}\vec{P}\right) \quad \text{and} \quad \vec{J}_{\text{eff}} = \frac{\chi-\chi_m}{\chi}\frac{\partial\vec{P}}{\partial t}$$

where χ and $\vec{P}$ are, respectively, the dielectric susceptibility and electric polarization inside the target, and χ_m is the dielectric susceptibility of water. Secondly, the following theorem of inverse scattering is proved: Let J_s (a four-component vector) represent the charge-current distribution of the scattering target, and A_s (also a 4-vector) the electric-magnetic potential of the scattered field; also, let J_w be an arbitrary localized 4-vector field and A_w be the 4-vector solution of the equation $(\nabla^2+k^2)A_w(\vec{x}) = -(4\pi/c)J_w(\vec{x})$, then

$$\iiint J_w(\vec{x})\cdot A_s(\vec{x})\,d\vec{x} = \iiint A_w(\vec{x})\cdot J_s(\vec{x})\,d\vec{x} \qquad (3)$$

where the products are the 4-vector scalar products and the integrations are over the entire space. Equation (3) turns out to be a very useful theorem for inverse scattering problems, especially in obtaining an image of a target from limited measurements of the scattered field. If measurements on $A_s(\vec{x})$ are carried out at a set of points $\{\vec{x}_n\}$, and one takes $J_w(\vec{x}) = \Sigma_n J_n\delta(\vec{x}-x_n)$, then the left-hand side of (3) is completely obtainable from the limited measurements. To be more explicit, if one measures only the y-polarization of the scattered electric field $\vec{E}_s(\vec{x})$ at $\{\vec{x}_n\}$, then take $\vec{J}_w = \hat{y}\Sigma_n J_n\delta(\vec{x}-\vec{x}_n)$, so (3) gives

$$\sum_n J_n E_y(\vec{x}_n) = \frac{c}{\epsilon_m}\iiint \vec{A}_w\cdot\left[k_m^2\left(\frac{\chi-\chi_m}{\chi}\right)\vec{P} + \nabla\left(\nabla\cdot\frac{\chi-\chi_m}{\chi}\vec{P}\right)\right]d\vec{x} \quad (4)$$

where $E_y(\vec{x}_n)$ is the measured y-component of the electric field at $\vec{x}_n$, k_m the wavenumber of the microwave in water, ϵ_m the dielectric constant of water, $\vec{P}$ the polarization inside the target induced by the incident wave, c the speed of light, and $\vec{A}_w(\vec{x})$ is given by $\Sigma_n\hat{y}J_n\exp(i|\vec{x}-\vec{x}_n|)/|\vec{x}-\vec{x}_n|$. It is possible to select a set of weights $\{J_n\}$ such that the resulting weighted field $A_w(\vec{x})$ in the region occupied by the target is very small, except for a spatially sharp peak at a focal point $\vec{x}_f$. Then the integral in the right-hand side of (4) is approximately equal to the value of

$$\frac{c}{\epsilon_m}\left[k_m^2\left(\frac{\chi-\chi_m}{\chi}\right)\vec{P} + \nabla\left(\nabla\cdot\frac{\chi-\chi_m}{\chi}\vec{P}\right)\right]$$

evaluated at $\vec{x}_f$, where $\vec{x}_f$ is a point inside the target.

The approach described above will require longer data processing time than the one proposed by Bolomey *et al.*, mainly because the calculation of the field $A_w(\vec{x})$ from $\{J_n\}$ is not a simple Fourier transform. It is hoped that some type of fast data processing algorithm similar to the fast Fourier transform may be developed in the near future for the purpose proposed here.

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Response to Reply to Comments on "Variational Methods for Nonstandard Eigenvalue Problems"

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In response to the author's reply to the first Comment[1] on the above paper,[2] it is affirmed that the proof of fallacy and statements contained therein stand unaltered. The author's attempt at

Manuscript received November 14, 1983.

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